Matrices and Determinants

Question1

If A is a square matrix such that $A^2 = A$, then $(I - A)^3$ is

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Options:

A. $\mathrm{I} - \mathrm{A}$

B. $A - I$

C. $\mathrm{I} + \mathrm{A}$

D. $-\mathrm{I} - \mathrm{A}$

Answer: A

Solution:

$$\begin{aligned} \mathrm{A}^2 = \mathrm{A}. \quad (\mathrm{I}-\mathrm{A})^3 &= (\mathrm{I}-\mathrm{A})\left(\mathrm{I}-2\,\mathrm{A}+\mathrm{A}^2\right) \\ &= (\mathrm{I}-\mathrm{A})(\mathrm{I}-2\,\mathrm{A}+\mathrm{A}) \\ &= (\mathrm{I}-\mathrm{A})(\mathrm{I}+\mathrm{A}) \\ &= \mathrm{I}-\mathrm{A}^2 = \mathrm{I}-\mathrm{A} \end{aligned}$$

Question2

If A and B are two matrices such that AB is an identity matrix and the order of matrix B is 3×4, then the order of matrix A is

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Options:

A. 3×4

B. 3×3

C. 4×3

D. 4×4

Answer: C

Solution:

Let's find the size of A step by step:

B has size 3×4.

For AB to be defined, if A is $m \times n$, then n must equal 3.

So A is $m \times 3$, and

AB has size $(m \times 3)(3 \times 4) = (m \times 4)$.

Since AB is an identity matrix, it must be square. Hence $m = 4$.

Therefore

A is 4×3.

Answer: Option C, 4×3.

Question3

Which of the following statements is not correct?

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Options:

A. A row matrix has only one row

B. A diagonal matrix has all diagonal elements equal to zero

C. A symmetric matrix A is a square matrix satisfying $A' = A$.

D. A skew symmetric matrix has all diagonal elements equal to zero

Answer: B

Solution:

A diagonal matrix need not be contains only zero as it diagonal element.

Question4

If a matrix $A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$ satisfies $A^6 = kA'$, then the value of k is

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Options:

A. 32

B. 1

C. $\frac{1}{32}$

D. 6

Answer: A

Solution:

$$A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix}$$

$$A^3 = \begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 4 & 4 \\ 4 & 4 \end{bmatrix} = \begin{bmatrix} 2^2 & 2^2 \\ 2^2 & 2^2 \end{bmatrix}$$

$$A^4 = \begin{bmatrix} 8 & 8 \\ 8 & 8 \end{bmatrix} = \begin{bmatrix} 2^3 & 2^3 \\ 2^3 & 2^3 \end{bmatrix}$$

$$A^6 = \begin{bmatrix} 2^5 & 2^5 \\ 2^5 & 2^5 \end{bmatrix} = 2^5 \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \Rightarrow 2^5 = 32$$

$$\therefore k = 32$$

Question5

If $A = \begin{bmatrix} k & 2 \\ 2 & k \end{bmatrix}$ and $|A^3| = 125$, then the value of k is

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Options:

A. ± 2

B. ± 3

C. -5

D. -4

Answer: B

Solution:

$$A=\begin{bmatrix} k & 2 \\ 2 & k \end{bmatrix}$$
$$|A|=k^2-4$$
$$|A|^3=|A|\cdot|A|\cdot|A|=125,\ \text{given}$$
$$\Rightarrow \left(k^2-4\right)^3=125$$
$$\Rightarrow k^2-4=5 \Rightarrow k^2=9$$
$$\Rightarrow k=\pm 3$$

Question6

If A is a square matrix satisfying the equation $A^2-5A+7I=0$, where I is the I dentity matrix and 0 is null matrix of same order, then $A^{-1}=$

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Options:

A. $\frac{1}{7}(5\mathrm{I}-\mathrm{A})$

B. $\frac{1}{7}(\mathrm{~A}-5\mathrm{I})$

C. $7(5\mathrm{I}-\mathrm{A})$

D. $\frac{1}{5}(7\mathrm{I}-\mathrm{A})$

Answer: A

Solution:

Given

$$\mathrm{A}^2-5\mathrm{~A}+7\mathrm{I}=0\ \left(\because \text{ Multiply by } \mathrm{A}^{-1} \text{ both sides } |\mathrm{A}| \neq 0\right)$$
$$\mathrm{A}-5\mathrm{I}+7\mathrm{~A}^{-1}=0$$
$$\mathrm{A}^{-1}=\frac{(5\mathrm{I}-\mathrm{A})}{7}$$

Question7

If A is a square matrix of order 3×3, $\det A = 3$, then the value of $\det\left(3A^{-1}\right)$ is

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Options:

A. $\frac{1}{3}$

B. 3

C. 27

D. 9

Answer: D

Solution:

First note for a 3×3 matrix A with $\det A = 3$:

$\det(A^{-1}) = \frac{1}{\det A} = \frac{1}{3}$.

When you multiply a 3×3 matrix by a scalar 3, its determinant is multiplied by 3^3. So

$\det\left(3A^{-1}\right) = 3^3\ \det(A^{-1}) = 27\ \times \frac{1}{3} = 9.$

Hence the correct choice is Option D: 9.

Question8

If $B = \begin{bmatrix} 1 & 3 \\ 1 & \alpha \end{bmatrix}$ be the adjoint of a matrix A and $|A| = 2$, then the value of α is

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Options:

A. 4

B. 5

C. 2

D. 3

Answer: B

Solution:

Let $B = \text{adj}(A)$ for a 2×2 matrix $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$. Recall that for 2×2,

$\text{adj}(A) = \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$.

Here we're given

$B = \begin{pmatrix} 1 & 3 \\ 1 & \alpha \end{pmatrix}$.

By matching entries:

- $d = 1$
- $-b = 3 \implies b = -3$
- $-c = 1 \implies c = -1$
- $a = \alpha$

So

$A = \begin{pmatrix} \alpha & -3 \\ -1 & 1 \end{pmatrix}, \quad |A| = \alpha \cdot 1 - (-3)(-1) = \alpha - 3.$

Since $|A| = 2$, we get

$\alpha - 3 = 2 \quad \implies \quad \alpha = 5.$

Answer: 5.

Question9

If A is a square matrix, such that $A^2 = A$, then $(I + A)^3$ is equal to

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Options:

A. $A - I$

B. $7A$

C. $7A + I$

D. $I - 7A$

Answer: C

Solution:

$$
\begin{aligned}
(I+A)^3 &= I^3 + A^3 + 3I^2A + 3A^2I \\
&= I + A^3 + 3A + 3A^2 \\
&= I + A^2 \cdot A + 3A + 3A \quad [\because A^2 = A] \\
&= I + A \cdot A + 6A \\
&= I + A^2 + 6A \\
&= I + A + 6A \quad [\because A^2 = A] \\
&= I + 7A
\end{aligned}
$$

Question10

If $A = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$, then A^{10} is equal to

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Options:

A. $2^8 A$

B. 2^9 A

C. $2^{10} A$

D. $2^{11} A$

Answer: B

Solution:

$$\because \quad A^2 = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$$

$$
\begin{aligned}
\therefore \quad A^2 &= \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} = \begin{pmatrix} 1+1 & 1+1 \\ 1+1 & 1+1 \end{pmatrix} \\
&= \begin{pmatrix} 2 & 2 \\ 2 & 2 \end{pmatrix} = 2 \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} = 2A
\end{aligned}
$$

$$
\begin{aligned}
\therefore \quad A^3 &= \begin{pmatrix} 2 & 2 \\ 2 & 2 \end{pmatrix} \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \\
&= \begin{pmatrix} 2+2 & 2+2 \\ 2+2 & 2+2 \end{pmatrix} = \begin{pmatrix} 4 & 4 \\ 4 & 4 \end{pmatrix} \\
&= 4 \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} = 4A
\end{aligned}
$$

Hence, $A^{10} = 2^9 A$

$[\because A^n = 2^{n-1} A]$

Question11

If $f(x)=\begin{vmatrix} x-3 & 2x^2-18 & 2x^3-81 \\ x-5 & 2x^2-50 & 4x^3-500 \\ 1 & 2 & 3 \end{vmatrix}$**, then** $f(1)\cdot f(3)+f(3)\cdot f(5)+f(5)\cdot f(1)$ **is**

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Options:

A. -1

B. 0

C. 1

D. 2

Answer: A

Solution:

$$f(x)=2(x-5)\begin{bmatrix} x-3 & x^2-9 & 2x^3-81 \\ 1 & x+5 & 4\left(x^2+5x+25\right) \\ 1 & 1 & \end{bmatrix}$$

$$\Rightarrow f(5)=0$$

$$\text{and } f(1)=2(-4)\begin{bmatrix} -2 & -8 & -79 \\ 1 & 6 & 124 \\ 1 & 1 & 3 \end{bmatrix}$$

$$=-8[212-968+395]=2888$$

$$f(3)=2(-2)\begin{bmatrix} 0 & 0 & -27 \\ 1 & 8 & 196 \\ 1 & 1 & 3 \end{bmatrix}$$

$$=-4[0+0-27(1-8)]=-756$$

$$\text{So, } f(1)\cdot f(3)+f(3)\cdot f(5)+f(5)\cdot f(1)$$

$$=2888\times(-756)+0+0=-2183328$$

No option matched.

Question12

If $P=\begin{bmatrix}1 & \alpha & 3\\ 1 & 3 & 3\\ 2 & 4 & 4\end{bmatrix}$ is the adjoint of a 3×3 matrix A and $|A|=4$, then α is equal to

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Options:

A. 4

B. 5

C. 11

D. 0

Answer: C

Solution:

$$P=\begin{bmatrix}1 & \alpha & 3\\ 1 & 3 & 3\\ 2 & 4 & 4\end{bmatrix}$$

For 3×3, matrix,

$$|\operatorname{adj} A|=|A|^2$$
$$|\operatorname{adj} A|=(4)^2=16$$

Now, $P=\begin{vmatrix}1 & \alpha & 3\\ 1 & 3 & 3\\ 2 & 4 & 4\end{vmatrix}=16$

$$\Rightarrow \quad 1(12-12)-\alpha(4-6)+3(4-6)=16$$
$$\Rightarrow \quad 0+2\alpha-6=16$$
$$\Rightarrow \quad 2\alpha=22$$

Hence, $\alpha=11$

Question13

If $A=\begin{vmatrix}x & 1\\ 1 & x\end{vmatrix}$ and $B=\begin{vmatrix}x & 1 & 1\\ 1 & x & 1\\ 1 & 1 & x\end{vmatrix}$, then $\frac{dB}{dx}$ is

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Options:

A. $3A$

B. $-3B$

C. $3B+1$

D. $1-3A$

Answer: A

Solution:

$$B=\begin{vmatrix} x & 1 & 1 \\ 1 & x & 1 \\ 1 & 1 & x \end{vmatrix}$$

$$=\begin{vmatrix} 1 & 1 & 1 \\ 0 & x & 1 \\ 0 & 1 & x \end{vmatrix}+\begin{vmatrix} x & 0 & 1 \\ 1 & 1 & 1 \\ 1 & 0 & x \end{vmatrix}+\begin{vmatrix} x & 1 & 0 \\ 1 & x & 0 \\ 1 & 1 & 1 \end{vmatrix}$$

$$=\left(x^2-1\right)+\left(x^2-1\right)+\left(x^2-1\right)$$

$$=3\left(x^2-1\right)$$

$$=3A \quad \left[\because A=\begin{vmatrix} x & 1 \\ 1 & x \end{vmatrix}=x^2-1\right]$$

Question14

The value of $\begin{vmatrix} \sin^2 14^\circ & \sin^2 66^\circ & \tan 135^\circ \\ \sin^2 66^\circ & \tan 135^\circ & \sin^2 14^\circ \\ \tan 135^\circ & \sin^2 14^\circ & \sin^2 66^\circ \end{vmatrix}$ **is**

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Options:

A. 0

B. 1

C. 2

D. -1

Answer: A

Solution:

Here, $\begin{vmatrix} \sin^2 14^\circ & \sin^2 66^\circ & \tan 135^\circ \\ \sin^2 66^\circ & \tan 135^\circ & \sin^2 14^\circ \\ \tan 135^\circ & \sin^2 14^\circ & \sin^2 66^\circ \end{vmatrix}$

Applying $C_1 \to C_1 + C_2 + C_3$

$$= \begin{vmatrix} \sin^2 14^\circ + \sin^2 66^\circ + \tan 135^\circ & \sin^2 66^\circ \tan 135^\circ \\ = \sin^2 14^\circ + \sin^2 66^\circ + \tan 135^\circ & \tan 135^\circ \sin^2 14^\circ \\ \sin^2 14^\circ + \sin^2 66^\circ + \tan 135^\circ & \sin^2 14^\circ \sin^2 66^\circ \end{vmatrix}$$

$$= \sin^2 14^\circ + \sin^2 66^\circ + \tan 135^\circ$$

$$\begin{vmatrix} 1 & \sin^2 66^\circ & \tan 135^\circ \\ 1 & \tan 135^\circ & \sin^2 14^\circ \\ 1 & \sin^2 14^\circ & \sin^2 66^\circ \end{vmatrix}$$

$$= \sin^2 14^\circ + \sin^2 66^\circ - 1 \begin{vmatrix} 1 & \sin^2 66 & -1 \\ 1 & -1 & \sin^2 14^\circ \\ 1 & \sin^2 14^\circ & \sin^2 66^\circ \end{vmatrix}.$$

Applying $R_1 \to R_1 - R_2, R_2 \to R_2 - R_3$

$$= \cos^2 66^\circ + \sin^2 66^\circ - 1$$

$$\begin{vmatrix} 0 & 1 + \sin^2 66^\circ & -1 - \sin^2 14^\circ \\ 0 & -1 - \sin^2 14^\circ & \sin^2 14^\circ - \sin^2 66^\circ \\ 1 & \sin^2 14^\circ & \sin^2 66^\circ \end{vmatrix}$$

$$\left(\cos^2 66^\circ + \sin^2 66^\circ - 1\right) = 0$$

Question15

If $x \begin{bmatrix} 3 \\ 2 \end{bmatrix} + y \begin{bmatrix} 1 \\ -1 \end{bmatrix} = \begin{bmatrix} 15 \\ 5 \end{bmatrix}$, then the value of x and y are

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Options:

A. $x = 4, y = -3$

B. $x = -4, y = -3$

C. $x = -4, y = 3$

D. $x = 4, y = 3$

Answer: D

Solution:

We have,

$$x\begin{bmatrix}3\\2\end{bmatrix}+y\begin{bmatrix}1\\-1\end{bmatrix}=\begin{bmatrix}15\\5\end{bmatrix}$$
$$\begin{bmatrix}3x\\2x\end{bmatrix}+\begin{bmatrix}y\\-y\end{bmatrix}=\begin{bmatrix}15\\5\end{bmatrix}$$
$$\begin{bmatrix}3x+y\\2x-y\end{bmatrix}=\begin{bmatrix}15\\5\end{bmatrix}$$
$$3x+y=15 \quad \text{..... (i)}$$
$$2x-y=5 \quad \text{..... (ii)}$$

Adding Eqs. (i) and (ii), we get

$5x=20 \Rightarrow x=4$

So, $y=15-3x$ [From Eq. (i)]
$y=15-3\times4=3$

Question16

If A and B are two matrices, such that $AB=B$ and $BA=A$, then A^2+B^2 equals to

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Options:

A. $2AB$

B. AB

C. $2BA$

D. $A+B$

Answer: D

Solution:

Given, $AB=B$ and $BA=A$

$$A^2+B^2=AA+BB=A(BA)+B(AB)$$
$$=(AB)(A)+(BA)B=BA+AB=A+B$$

Question17

If $A=\begin{bmatrix}2-k & 2\\ 1 & 3-k\end{bmatrix}$ is singular matrix, then the value of $5k-k^2$ is equal to

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Options:

A. -6

B. -4

C. 6

D. 4

Answer: D

Solution:

$A=\begin{bmatrix}2-k & 2\\ 1 & 3-k\end{bmatrix}$ is singular matrix we know that A is singular matrix, then $|A|=0$

$$
\begin{aligned}
& |A|=(2-k)(3-k)-2 \\
\Rightarrow \quad & 0=6-2k-3k+k^2-2 \\
\Rightarrow \quad & k^2-5k+4=0 \Rightarrow k^2-5k=-4 \\
\Rightarrow \quad & 5k-k^2=4
\end{aligned}
$$

Question18

If $\Delta=\begin{vmatrix}1 & a & a^2\\ 1 & b & b^2\\ 1 & c & c^2\end{vmatrix}$ and $\Delta_1=\begin{vmatrix}1 & 1 & 1\\ bc & ca & ab\\ a & b & c\end{vmatrix}$, then

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Options:

A. $\Delta_1=3\Delta$

B. $\Delta_1 \neq \Delta$

C. $\Delta_1 = -\Delta$

D. $\Delta_1 = \Delta$

Answer: C

Solution:

Given,

$$\Delta = \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}$$

$$\text{and } \Delta_1 = \begin{vmatrix} 1 & 1 & 1 \\ bc & ca & ab \\ a & b & c \end{vmatrix}$$

$$\Delta = \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}$$

$R_1 \to R_1 - R_2$ and $R_2 \to R_2 - R_3$

$$\Delta = \begin{vmatrix} 0 & a-b & a^2-b^2 \\ 0 & b-c & b^2-c^2 \\ 1 & c & c^2 \end{vmatrix}$$

$$\Delta = (a-b)(b-c) \begin{vmatrix} 0 & 1 & a+b \\ 0 & 1 & b+c \\ 1 & c & c^2 \end{vmatrix}$$

$$\Delta = (a-b)(b-c)(c-a) \quad \text{.... (i)}$$

$$\text{Now, } \Delta_1 = \begin{vmatrix} 1 & 1 & 1 \\ bc & ca & ab \\ a & b & c \end{vmatrix}$$

$C_1 \to C_1 - C_2, C_2 \to C_2 - C_3$

$$\Delta_1 = \begin{vmatrix} 0 & 0 & 1 \\ c(b-a) & a(c-b) & ab \\ a-b & b-c & c \end{vmatrix}$$

$$\Delta_1 = (a-b)(b-c) \begin{vmatrix} 0 & 0 & 1 \\ -c & -a & ab \\ 1 & 1 & c \end{vmatrix}$$

$$\Delta_1 = (a-b)(b-c)(a-c)$$

$$\Delta_1 = -(a-b)(b-c)(c-a) \quad \text{.... (ii)}$$

From Eqs. (i) and (ii), we get

$\Delta = -\Delta_1$

Question19

If $A = \begin{bmatrix} 1 & \tan \alpha/2 \\ -\tan \alpha/2 & 1 \end{bmatrix}$ and $AB = I$, then B is equal to

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Options:

A. $\cos^2 \alpha/2 \cdot A$

B. $\cos^2 \alpha/2 \cdot I$

C. $\sin^2 \alpha/2 \cdot A$

D. $\cos^2 \alpha/2 \cdot A^T$

Answer: D

Solution:

Given, $A = \begin{bmatrix} 1 & \tan \frac{\alpha}{2} \\ -\tan \frac{\alpha}{2} & 1 \end{bmatrix}$ and $AB = I$

$\Rightarrow B = A^{-1}$

Now $A = \begin{bmatrix} 1 & \tan \frac{\alpha}{2} \\ -\tan \frac{\alpha}{2} & 1 \end{bmatrix}$

$|A| = \begin{bmatrix} 1 & \tan \frac{\alpha}{2} \\ -\tan \frac{\alpha}{2} & 1 \end{bmatrix} = 1 + \operatorname{an}^2 \frac{\alpha}{2} = \sec^2 \frac{\alpha}{2}$

$\operatorname{adj} A = \begin{bmatrix} 1 & -\tan \frac{\alpha}{2} \\ \tan \frac{\alpha}{2} & 1 \end{bmatrix}$

$A^{-1} = \frac{1}{\sec^2 \frac{\alpha}{2}} \begin{bmatrix} 1 & -\tan \frac{\alpha}{2} \\ \tan \frac{\alpha}{2} & 1 \end{bmatrix}$

$= \cos^2 \frac{\alpha}{2} \begin{bmatrix} 1 & -\tan \frac{\alpha}{2} \\ \tan \frac{\alpha}{2} & 1 \end{bmatrix} = \cos^2 \frac{\alpha}{2} AT$

$B = \left[\cos^2 \frac{\alpha}{2}\right] AT$

Question20

If A is a matrix of order 3×3, then $\left(A^2\right)^{-1}$ is equal to

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Options:

A. $\left(-A^2\right)^2$

B. $\left(A^{-1}\right)^2$

C. A^2

D. $(-A)^{-2}$

Answer: B

Solution:

$$\left(A^2\right)^{-1}=(A)^{-2}=\left(A^{-1}\right)^2=\left(-A^{-1}\right)^2=(-A)^{-2}$$
$$\left(A^2\right)^{-1}\neq A^2,\left(-A^2\right)^2$$

Question21

If $A=\begin{bmatrix}2 & -1\\ 3 & -2\end{bmatrix}$, then the inverse of the matrix A^3 is

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Options:

A. A

B. -1

C. 1

D. $-A$

Answer: A

Solution:

$$A=\begin{bmatrix}2 & -1\\ 3 & -2\end{bmatrix}$$
$$|A|=-4+3=-1$$
$$\operatorname{adj}(A)=\begin{bmatrix}-2 & -3\\ -(-1) & 2\end{bmatrix}^T=\begin{bmatrix}-2 & -3\\ 1 & 2\end{bmatrix}^T=\begin{bmatrix}-2 & 1\\ -3 & 2\end{bmatrix}$$
$$A^{-1}=\frac{\operatorname{adj}(A)}{|A|}=\frac{\begin{bmatrix}-2 & 1\\ -3 & 2\end{bmatrix}}{(-1)}=\begin{bmatrix}2 & -1\\ 3 & -2\end{bmatrix}=A$$

$$\Rightarrow \quad A^{-1}=A \Rightarrow A\cdot A^{-1}=A\cdot A$$
$$\Rightarrow \quad I=A^2 \Rightarrow A\cdot I=A\cdot A^2$$
$$\Rightarrow \quad A=A^3 \Rightarrow (A)^{-1}=\left(A^3\right)^{-1}$$
$$\Rightarrow \quad A=\left(A^3\right)^{-1} \Rightarrow \left(A^3\right)^{-1}=A$$

Question22

If A is a skew symmetric matrix, then A^{2021} is

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Options:

A. Row matrix

B. Column matrix

C. Symmetric matrix

D. Skew symmetric matrix

Answer: D

Solution:

Given, $A^T=-A$

Let $\quad P=A^{2021}$

$$P^T=\left[A^{2021}\right]^T=\left[A^T\right]^{2021}$$
$$=[-A]^{2021}=-[A]^{2021}=-P$$

Hence, A^{2021} is also a skew symmetric matrix.

Question23

If $A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$, then $(aI + bA)^n$ is (where I is the identify matrix of order 2)

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Options:

A. $a^2 I + a^{n-1} b \cdot A$

B. $a^n I + n \cdot a^{n-1} b \cdot A$

C. $a^n I + n a^n b A$

D. $a^n I + b^n A$

Answer: B

Solution:

Given, $A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$

Let $P(n) : (aI + bA)^n = a^n I + n a^{n-1} bA$

For $n = 1, p(1) : (aI + bA)^1 = a^1 I + 1 \cdot a^{1-1} bA$

$aI + bA = aI + bA$

It is true for $n = 1$.

Let $P(n)$ is true for $n = K$.

So, $P(K) : (AI + bA)^K = a^K I + K a^{K-1} bA$ .... (i)

Now, for $n = K + 1$

$P(K+1) : (aI + bA)^{K+1} = a^{K+1} I + (K+1) a^{K+1-1} bA$ .... (ii)

$$\begin{aligned} \text{LHS } &= (aI + bA)^{K+1} = (aI + bA)^K (aI + bA) \\ &= \left(a^K I + K a^{K-1} bA\right)(aI + bA) \quad [\text{ from Eq. (i)}] \\ &= a^{K+1} I + K a^K IbA + a^K IbA + K a^{K-1} b^2 A^2 \\ &= a^{K+1} I + K a^K b(IA) + a^K b(IA) + K a^{K-1} b^2 \left(A^2\right) \\ &= a^{K+1} I + K a^K bA + a^K bA + K a^{K-1} b^2 \times 0 \end{aligned}$$

[$\because IA = A$ and

$$\left. A^2 = A \cdot A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = 0 \right]$$

$= a^{K+1} I + (K+1) a^K bA =$ RHS

Hence, $P(n)$ is true for all $n \in N$.

Question24

If A is a 3×3 matrix such that $|5 \cdot \text{adj } A| = 5$, then $|A|$ is equal to

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Options:

A. ± 1

B. $\pm 1/25$

C. $\pm 1/5$

D. ± 5

Answer: C

Solution:

Given, A is a 3×3 matrix and $|5 \cdot \text{Adj}(A)| = 5$

$$|5 \cdot \text{adj}(A)| = 5 \Rightarrow 5^3 |\text{adj}(A)| = 5$$
$$\Rightarrow \quad |\text{adj}(A)| = \frac{1}{5^2} \Rightarrow |A|^{3-1} = \frac{1}{5^2}$$
$$\Rightarrow \quad |A|^2 = \left(\frac{1}{5}\right)^2 \Rightarrow |A| = \pm\frac{1}{5}$$

Question25

If there are two values of 'a' which makes determinant

$$\Delta = \begin{vmatrix} 1 & -2 & 5 \\ 2 & a & -1 \\ 0 & 4 & 2a \end{vmatrix} = 86$$

Then, the sum of these number is

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Options:

A. -4

B. 9

C. 4

D. 5

Answer: A

Solution:

Given, $\Delta = \begin{vmatrix} 1 & -2 & 5 \\ 2 & a & -1 \\ 0 & 4 & 2a \end{vmatrix} = 86 \Rightarrow$

$$\begin{vmatrix} 1 & -2 & 5 \\ 2 & a & -1 \\ 0 & 4 & 2a \end{vmatrix} = 86$$
$$\Rightarrow 1\left[2a^2 + 4\right] + 2[4a + 0] + 5[8 - 0] = 86$$
$$\Rightarrow \quad 2a^2 + 4 + 8a + 40 = 86$$
$$\Rightarrow \quad 2a^2 + 8a - 42 = 0 \Rightarrow a^2 + 4a - 21 = 0$$
$$\Rightarrow \quad a^2 + 7a - 3a - 21 = 0 \Rightarrow (a+7)(a-3) = 0$$
$$a = -7, 3$$

$\therefore$ Required sum $= -7 + 3 = -4$

Question26

If $A_n = \begin{bmatrix} 1-n & n \\ n & 1-n \end{bmatrix}$, then $|A_1| + |A_2| + \ldots . |A_{2021}| =$

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Options:

A. -2021

B. $-(2021)^2$

C. $(2021)^2$

D. 4042

Answer: B

Solution:

Given,

$$A_n = \begin{bmatrix} 1-n & n \\ n & 1-n \end{bmatrix} \text{ and}$$

$$|A_n| = \begin{bmatrix} 1-n & n \\ n & 1-n \end{bmatrix}$$
$$= (1-n)^2 - n^2 = 1 + n^2 - 2n - n^2$$
$$|A_n| = 1 - 2n$$

Now, $|A_1| + |A_2| + |A_3| + \ldots + |A_{2021}|$
$$= (1-2) + (1-4) + (1-6) + \ldots + (1-4042)$$
$$= (1+1+\ldots+1) - (2+4+6+\ldots+4042)$$
$$= 2021 - \left[\frac{2021}{2}(2+4042)\right]$$
$$= 2021 - \frac{2021}{2} \times 4044 = 2021 - 2021 \times 2022$$
$$= 2021(1-2022) = -(2021)^2$$

Question27

If $A = \begin{bmatrix} 1 & -2 & 1 \\ 2 & 1 & 3 \end{bmatrix}$

$B = \begin{bmatrix} 2 & 1 \\ 3 & 2 \\ 1 & 1 \end{bmatrix}$**, then** $(AB)'$ **is equal to**

KCET 2021

Options:

A. $\begin{bmatrix} -3 & -2 \\ 10 & 7 \end{bmatrix}$

B. $\begin{bmatrix} -3 & 10 \\ -2 & 7 \end{bmatrix}$

C. $\begin{bmatrix} -3 & 7 \\ 10 & 2 \end{bmatrix}$

D. $\begin{bmatrix} -3 & 7 \\ 10 & -2 \end{bmatrix}$

Answer: B

Solution:

Given, $A=\begin{bmatrix}1 & -2 & 1\\ 2 & 1 & 3\end{bmatrix} B=\begin{bmatrix}2 & 1\\ 3 & 2\\ 1 & 1\end{bmatrix}$

$$AB=\begin{bmatrix}1 & -2 & 1\\ 2 & 1 & 3\end{bmatrix}\begin{bmatrix}2 & 1\\ 3 & 2\\ 1 & 1\end{bmatrix}$$

$$=\begin{bmatrix}2-6+1 & 1-4+1\\ 4+3+3 & 2+2+3\end{bmatrix}$$

$$=\begin{bmatrix}-3 & -2\\ 10 & 7\end{bmatrix}$$

$$AB'=\begin{bmatrix}-3 & 10\\ -2 & 7\end{bmatrix}$$

Question28

Let M be 2×2 symmetric matrix with integer entries, then M is invertible if

KCET 2021

Options:

A. the first column of M is the transpose of second row of M.

B. the second row of M is the transpose of first column of M.

C. M is diagonal matrix with non-zero entries in the principal diagonal.

D. The product of entries in the principal diagonal of M is the product of entries in the other diagonal.

Answer: C

Solution:

Let a symmetric matrix

$$M=\begin{bmatrix}a & c\\ c & b\end{bmatrix}$$

For matrix to be invertible, determinant must not be equal to zero.

$$|M|=ab-c^2\neq 0$$
$$\Rightarrow \quad ab\neq c^2$$

Therefore, M is a diagonal matrix with non-zero entries in the main diagonal and the product of entries in the main diagonal of M is not the square of an integer.

Question29

If A and B are matrices of order 3 and $|A| = 5, |B| = 3$, then $|3AB|$ is

KCET 2021

Options:

A. 425

B. 405

C. 565

D. 585

Answer: B

Solution:

Given order of matrix A and B are 3,

$|A| = 5, |B| = 3$

$\Rightarrow$ The order of AB matrix is also 3.

$|AB| = |A| \cdot |B| = 15$

Using the property $|KA| = K^n|A|$, where n is the order of square matrix

$$\begin{aligned} \Rightarrow \quad |3AB| &= 3^3|AB| \\ &= 27 \times 15 = 405 \end{aligned}$$

Question30

If A and B are invertible matrices then which of the following is not correct?

KCET 2021

Options:

A. adj $A = |A| A^{-1}$

B. $\det\left(A^{-1}\right) = [\det(A)]^{-1}$

C. $(AB)^{-1} = B^{-1} A^{-1}$

D. $(A+B)^{-1} = B^{-1} + A^{-1}$

Answer: D

Solution:

If A and B are invertible matrices, then

(a) $(AB)^{-1} = B^{-1} A^{-1}$

(b) adj $A = |A| \cdot A^{-1}$

(c) $\left|A^{-1}\right| = |A|^{-1}$

Hence, $(A+B)^{-1} \neq B^{-1} + A^{-1}$

Question31

If $x^3 - 2x^2 - 9x + 18 = 0$ and $A = \begin{vmatrix} 1 & 2 & 3 \\ 4 & x & 6 \\ 7 & 8 & 9 \end{vmatrix}$ then the maximum value of A is

KCET 2021

Options:

A. 96

B. 36

C. 24

D. 120

Answer: A

Solution:

Given, $x^3 - 2x^2 - 9x + 18 = 0$ and $A = \begin{vmatrix} 1 & 2 & 3 \\ 4 & x & 6 \\ 7 & 8 & 9 \end{vmatrix}$

$$\begin{aligned} & x^3 - 2x^2 - 9x + 18 && = 0 \\ \Rightarrow \quad & x^2(x-2) - 9(x-2) && = 0 \\ \Rightarrow \quad & \left(x^2 - 9\right)(x-2) && = 0 \\ \Rightarrow \quad & (x-3)(x+3)(x-2) && = 0 \\ \therefore \quad & x = 2, 3, -3 \end{aligned}$$

$$\begin{aligned} A = & \begin{vmatrix} 1 & 2 & 3 \\ 4 & x & 6 \\ 7 & 8 & 9 \end{vmatrix} \\ & = 1(9x - 48) - 2(36 - 42 + 3(32 - 7x) \\ & = 9x - 48 + 12 + 96 - 21x \\ & = -12x + 60 \end{aligned}$$

$A(\text{when } x = 2) = -12 \times 2 + 60 = 36$

$A(\text{ when } x = 3) = -12 \times 3 + 60 = 24$

$A(\text{ when } x = -3) = -12 \times (-3) + 60 = 96$

More value of A at $x = -3$ is 96.

Question32

If $A = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$ then A^4 is equal to

KCET 2020

Options:

A. A

B. 2A

C. I

D. 4A

Answer: C

Solution:

We have,

$$A=\begin{bmatrix}0&0&1\\0&1&0\\1&0&0\end{bmatrix}$$

$$A^2=\begin{bmatrix}0&0&1\\0&1&0\\1&0&0\end{bmatrix}\begin{bmatrix}0&0&1\\0&1&0\\1&0&0\end{bmatrix}=\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}$$

$$A^4=A^2\cdot A^2=\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}=\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}=I$$

Question33

If $\begin{pmatrix}2&1\\3&2\end{pmatrix}A=\begin{pmatrix}1&0\\0&1\end{pmatrix}$ then the matrix a is

KCET 2020

Options:

A. $\begin{pmatrix}2&1\\3&2\end{pmatrix}$

B. $\begin{pmatrix}2&-1\\-3&2\end{pmatrix}$

C. $\begin{pmatrix}-2&1\\3&-2\end{pmatrix}$

D. $\begin{pmatrix}2&-1\\3&2\end{pmatrix}$

Answer: B

Solution:

We have,

$$\begin{bmatrix}2&1\\3&2\end{bmatrix}A=\begin{bmatrix}1&0\\0&1\end{bmatrix}$$

Let

$$B=\begin{bmatrix}2&1\\3&2\end{bmatrix}$$

$\therefore \quad BA = I$

$A = B^{-1}I$

$A = B^{-1}$

$\therefore \quad B^{-1} = \begin{bmatrix} 2 & -1 \\ -3 & 2 \end{bmatrix}$

Hence, $A = \begin{bmatrix} 2 & -1 \\ -3 & 2 \end{bmatrix}$

Question34

If $f(x) = \begin{vmatrix} x^3 - x & a + x & b + x \\ x - a & x^2 - x & c + x \\ x - b & x - c & 0 \end{vmatrix}$, then

KCET 2020

Options:

A. $f(1) = 0$

B. $f(2) = 0$

C. $f(0) = 0$

D. $f(-1) = 0$

Answer: C

Solution:

We have,

$$f(x) = \begin{vmatrix} x^3 - x & a + x & b + x \\ x - a & x^2 - x & c + x \\ x - b & x - c & 0 \end{vmatrix}$$

$$f(0) = \begin{vmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{vmatrix}$$

$f(0) = 0$

$f(0)$ is skew symmetric matrix of order 3.

Question35

If A and B are square matrices of same order and B is a skew symmetric matrix, then $A'BA$ is

KCET 2020

Options:

A. Symmetric matrix

B. Null matrix

C. Diagonal matrix

D. Skew symmetric matrix

Answer: B

Solution:

Given B is skew-symmetric matrix

$$\therefore \quad B' = -B$$
$$(A'BA)' = A'B'A = A'(-B)A = -(A'BA)$$

$\therefore A'BA$ is skew symmetric matrix.

Question36

If A is a square matrix of order 3 and $|A| = 5$, then $|\ A$ adj. $A\ |$ is

KCET 2020

Options:

A. 5

B. 125

C. 25

D. 625

Answer: B

Solution:

Given, $|A|=5$

$|\text{Aadj } A|=|A|^3$

$|\text{Aadj } A|=(5)^3=125$

Question37

If $a_1 a_2 a_3 \ldots a_9$ are in AP, then the value of $\begin{vmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 \end{vmatrix}$ is

KCET 2020

Options:

A. $\frac{9}{2}(a_1+a_9)$

B. (a_1+a_9)

C. $\log_e(\log_e e)$

D. 1

Answer: C

Solution:

$a_1, a_2, a_3, \ldots, a_9$ are in AP.

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 \end{vmatrix}$$

Apply $R_1 \rightarrow R_1+R_2-2R_2$

$$\begin{vmatrix} a_1+a_7-2a_4 & a_2+a_8-2a_5 & a_3+a_9-2a_6 \\ a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 \end{vmatrix}$$

$$\begin{vmatrix} 0 & 0 & 0 \\ a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 \end{vmatrix}$$

$[\because a_1+a_7-2a_4=a_1+a_1+6d-2a_1-6d=0$

Similarly $a_2+a_4-2a_5=0, a_3+a_9-2a_6=0]$

$=0$

$=\log_e 1=\log_e(\log_e e)$

Question38

The inverse of the matrix $\begin{bmatrix} 2 & 5 & 0 \\ 0 & 1 & 1 \\ -1 & 0 & 3 \end{bmatrix}$ **is**

KCET 2019

Options:

A. $\begin{bmatrix} 3 & -15 & 5 \\ -1 & 6 & -2 \\ 1 & -5 & 2 \end{bmatrix}$

B. $\begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}$

C. $\begin{bmatrix} 3 & -15 & 5 \\ -1 & 6 & -2 \\ 1 & -5 & -2 \end{bmatrix}$

D. $\begin{bmatrix} 3 & -5 & 5 \\ -1 & -6 & -2 \\ 1 & -5 & 2 \end{bmatrix}$

Answer: A

Solution:

Key Idea Firstly write the given matrix in the form of $A = IA$ then apply sequence of elementary row operation on A of LHS to convert it into identity matrix and same operations apply on I of RHS

$$\text{Let } A = \begin{bmatrix} 2 & 5 & 0 \\ 0 & 1 & 1 \\ -1 & 0 & 3 \end{bmatrix}$$

For applying row operations $A = IA$, where A is an identity matrix of order 3.

$$\text{i.e. } \begin{bmatrix} 2 & 5 & 0 \\ 0 & 1 & 1 \\ -1 & 0 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

On applying $R_1 \rightarrow \frac{R_1}{2}$, we get

$$\begin{bmatrix} 1 & 5/2 & 0 \\ 0 & 1 & 1 \\ -1 & 0 & 3 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

On applying $R_3 \to R_3 + R_1$,

$$\begin{bmatrix} 1 & 5/2 & 0 \\ 0 & 1 & 1 \\ 0 & 5/2 & 3 \end{bmatrix} = \begin{bmatrix} 1/2 & 0 & 0 \\ 0 & 1 & 0 \\ 1/2 & 0 & 1 \end{bmatrix} A$$

On applying $R_1 \to R_1 - \frac{5}{2} R_2, R_3 \to R_3 - \frac{5}{2} R_2$, we get

$$\begin{bmatrix} 1 & 0 & -5/2 \\ 0 & 1 & 1 \\ 0 & 0 & 1/2 \end{bmatrix} = \begin{bmatrix} 1/2 & -5/2 & 0 \\ 0 & 1 & 0 \\ 1/2 & -5/2 & 1 \end{bmatrix} A$$

On applying $R_3 \to 2R_3$, we get

$$\begin{bmatrix} 1 & 0 & -5/2 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1/2 & -5/2 & 0 \\ 0 & 1 & 0 \\ 1 & -5 & 2 \end{bmatrix} A$$

$R_1 \to R_1 + \frac{5}{2} R_3, R_2 \to R_2 - R_3$, we get

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & -15 & 5 \\ -1 & 6 & -2 \\ 1 & -5 & 2 \end{bmatrix} A$$

Hence, $A^{-1} = \begin{bmatrix} 3 & -15 & 5 \\ -1 & 6 & -2 \\ 1 & -5 & 2 \end{bmatrix}$

Question39

If P and Q are symmetric matrices of the same order then $PQ - QP$ is

KCET 2019

Options:

A. zero matrix

B. identity matrix

C. skew-symmetric matrix

D. symmetric matrix

Answer: C

Solution:

Since P and Q are symmetric matrices

$\therefore \quad P' = P$ and $Q' = Q$

Let $\quad A = PQ - QP$

$\Rightarrow \quad A' = (PQ - QP)' = (PQ)' - (QP)'$

$[\because (A+B)' = A' + B'; (AB)' = B'A']$

$= Q'P' - P'Q' = QP - PQ$

$= -(PQ - QP)$

$\Rightarrow \quad A' = -A$

$\Rightarrow PQ - QP$ is skew symmetric matrix.

Question40

If $3A + 4B' = \begin{bmatrix} 7 & -10 & 17 \\ 0 & 6 & 31 \end{bmatrix}$ **and** $2B + 3A' \begin{bmatrix} -1 & 18 \\ 4 & 0 \\ -5 & -7 \end{bmatrix}$ **then** $B =$

KCET 2019

Options:

A. $\begin{bmatrix} -1 & -18 \\ 4 & -16 \\ -5 & -7 \end{bmatrix}$

B. $\begin{bmatrix} 1 & 3 \\ -1 & 1 \\ 2 & 4 \end{bmatrix}$

C. $\begin{bmatrix} 1 & 3 \\ -1 & 1 \\ 2 & -4 \end{bmatrix}$

D. $\begin{bmatrix} 1 & -3 \\ -1 & 1 \\ 2 & 4 \end{bmatrix}$

Answer: B

Solution:

We have,

$$3A + 4B' = \begin{bmatrix} 7 & -10 & 17 \\ 0 & 6 & 31 \end{bmatrix} \text{ .... (i)}$$

$$\text{and } 2B - 3A' = \begin{bmatrix} -1 & 18 \\ 4 & 0 \\ -5 & -7 \end{bmatrix}$$

$$\Rightarrow 2B' - 3A = \begin{bmatrix} -1 & 4 & -5 \\ 18 & 0 & -7 \end{bmatrix} \text{ .... (ii)}$$

$(\because (A')' = A)$

adding Eqs. (i) and (ii), we get

$$6B' = \begin{bmatrix} 6 & -6 & 12 \\ 18 & 6 & 24 \end{bmatrix} \Rightarrow 6B = \begin{bmatrix} 6 & 18 \\ -6 & 6 \\ 12 & 24 \end{bmatrix}$$

$$\Rightarrow \quad B = \begin{bmatrix} 1 & 3 \\ -1 & 1 \\ 2 & 4 \end{bmatrix}$$

Question41

If $A = \begin{bmatrix} 1 & 3 \\ 4 & 2 \end{bmatrix}, B = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$, Then $|ABB'| =$

KCET 2019

Options:

A. 100

B. 50

C. 250

D. -250

Answer: D

Solution:

We have, $A=\begin{bmatrix}1 & 3\\ 4 & 2\end{bmatrix}$

$\Rightarrow \quad |A|=2-12=-10$

and $B=\begin{bmatrix}2 & -1\\ 1 & 2\end{bmatrix} \Rightarrow B'=\begin{bmatrix}2 & 1\\ -1 & 2\end{bmatrix}$

$\Rightarrow \quad |B|=|B'|=4+1=5$

Now, $|ABB'|=|A|\,|BB'|=|A||B|\,|B'|$

$=(-10)(5)(5)=-250$

Question42

If the value of a third order determinant is 16, then the value of the determinant formed by replacing each of its elements by its cofactor is

KCET 2019

Options:

A. 256

B. 96

C. 16

D. 48

Answer: A

Solution:

Let $|A|=16$

$\operatorname{adj}(A)=$ Transpose of co-factors of matrix A

$\Rightarrow$ Transpose of co-factor matrix $A|=|$ adj A $|$

$=|A|^{3-1}=|A|^2=(16)^2=256$

$\Rightarrow|$ co-factors matrix of A $|=256$

Question43

The constant term in the expansion of $\begin{vmatrix} 3x+1 & 2x-1 & x+2 \\ 5x-1 & 3x+2 & x+1 \\ 7x-2 & 3x+1 & 4x-1 \end{vmatrix}$ **is**

KCET 2019

Options:

A. -10

B. 0

C. 6

D. 2

Answer: C

Solution:

Given, $A = \begin{vmatrix} 3x+1 & 2x-1 & x+2 \\ 5x-1 & 3x+2 & x+1 \\ 7x-2 & 3x+1 & 4x-1 \end{vmatrix}$

for constant term, we put $x = 0$ in Δ, We get,

$$\Delta = \begin{vmatrix} 1 & -1 & 2 \\ -1 & 2 & 1 \\ -2 & 1 & -1 \end{vmatrix}$$
$$= 1(-2-1) + 1(1+2) + 2(-1+4)$$
$$= -3 + 3 + 6 = 6$$

Question44

If $A = \begin{bmatrix} 2 & -2 \\ -2 & 2 \end{bmatrix}$**, then** $A^n = 2^k A$**, where** k **is equal to**

KCET 2018

Options:

A. 2^{n-1}

B. $n+1$

C. $n-1$

D. $2(n-1)$

Answer: D

Solution:

Given, $A=\begin{bmatrix}2 & -2\\ -2 & 2\end{bmatrix}$

Now,

$$A^2=\begin{bmatrix}2 & -2\\ -2 & 2\end{bmatrix}\begin{bmatrix}2 & -2\\ -2 & 2\end{bmatrix}$$

$$\Rightarrow \quad A^2=\begin{bmatrix}8 & -8\\ -8 & 8\end{bmatrix}$$

$$\Rightarrow \quad A^2=2^2\begin{bmatrix}2 & -2\\ -2 & 2\end{bmatrix}$$

$$\Rightarrow \quad A^2=2^2A$$

Now, $\quad A^3=A^2\cdot A$

$$\begin{aligned}
&\Rightarrow \quad A^3=2^2\cdot A^2\\
&\Rightarrow \quad A^3=2^4\cdot A\\
&\Rightarrow \quad A^4=A^3\cdot A\\
&\Rightarrow \quad A^4=2^4\cdot A^2\\
&\Rightarrow \quad A^4=2^6A\\
&\Rightarrow \quad A^5=A^4\cdot A\\
&\Rightarrow \quad A^5=2^6A^2\\
&\qquad\ \ A^5=2^8A\\
&\therefore \quad A^n=2^{2(n-1)}A
\end{aligned}$$

Hence, $k=2(n-1)$

Question45

If $\begin{bmatrix}1 & 1\\ -1 & 1\end{bmatrix}\begin{bmatrix}x\\ y\end{bmatrix}=\begin{bmatrix}2\\ 4\end{bmatrix}$, then the values of x and y respectively are

KCET 2018

Options:

A. $-3,-1$

B. 1,3

C. 3,1

D. $-1,3$

Answer: D

Solution:

To find the values of x and y, we start with the matrix equation:

$$\begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 \\ 4 \end{bmatrix}$$

We can solve for the vector $\begin{bmatrix} x \\ y \end{bmatrix}$ using the inverse of the coefficient matrix:

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}^{-1} \begin{bmatrix} 2 \\ 4 \end{bmatrix}$$

The inverse of the matrix $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is given by:

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} = \frac{1}{ad-bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

For our matrix $\begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}$, the determinant $ad - bc = (1)(1) - (1)(-1) = 2$.

Thus, the inverse is:

$$\frac{1}{2} \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$$

Substitute the inverse into the expression for $\begin{bmatrix} x \\ y \end{bmatrix}$:

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{2} \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 4 \end{bmatrix}$$

Calculate the matrix product:

$$\frac{1}{2} \begin{bmatrix} (1)(2) + (-1)(4) \\ (1)(2) + (1)(4) \end{bmatrix} = \frac{1}{2} \begin{bmatrix} 2 - 4 \\ 2 + 4 \end{bmatrix} = \frac{1}{2} \begin{bmatrix} -2 \\ 6 \end{bmatrix}$$

This simplifies to:

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -1 \\ 3 \end{bmatrix}$$

Therefore, the values of x and y are $x = -1$ and $y = 3$.

Question46

If $A = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$, then AA' is equal to

KCET 2018

Options:

A. A

B. zero matrix

C. A'

D. 1

Answer: D

Solution:

Given the matrix $A = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$, we need to calculate the product AA', where A' is the transpose of matrix A.

The transpose of A, denoted A', is:

$$A' = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$$

Now, we calculate the product AA':

$$AA' = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix} \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$$

Performing the matrix multiplication, we have:

$$AA' = \begin{bmatrix} (\cos\alpha)(\cos\alpha) + (\sin\alpha)(\sin\alpha) & (\cos\alpha)(-\sin\alpha) + (\sin\alpha)(\cos\alpha) \\ (-\sin\alpha)(\cos\alpha) + (\cos\alpha)(\sin\alpha) & (-\sin\alpha)(-\sin\alpha) + (\cos\alpha)(\cos\alpha) \end{bmatrix}$$

Calculating each element:

Top left: $\cos^2\alpha + \sin^2\alpha = 1$ (using the Pythagorean identity)

Top right: $-\cos\alpha\sin\alpha + \sin\alpha\cos\alpha = 0$

Bottom left: $-\sin\alpha\cos\alpha + \cos\alpha\sin\alpha = 0$

Bottom right: $\sin^2\alpha + \cos^2\alpha = 1$ (using the Pythagorean identity)

Thus, the product is:

$$AA' = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

This is the identity matrix I. Hence,

$AA' = I$

Question47

If $x, y, z \in R$, then the value of determinant
$$\begin{vmatrix} (5^x + 5^{-x})^2 & (5^x - 5^{-x})^2 & 1 \\ (6^x + 6^{-x})^2 & (6^x - 6^{-x})^2 & 1 \\ (7^x + 7^{-x})^2 & (7^x - 7^{-x})^2 & 1 \end{vmatrix}$$ **is**

KCET 2018

Options:

A. 10

B. 12

C. 1

D. 0

Answer: D

Solution:

Let

$$A=\begin{vmatrix}(5^x+5^x)^2 & (5^x-5^x)^2 & 1\\(6^x+6^{-x})^2 & (6^x-6^{-x})^2 & 1\\(7^x+7^{-x})^2 & (7^x-7^{-x})^2 & 1\end{vmatrix}$$

$$C_1\to C_1-C_2$$

$$A=\begin{vmatrix}(5^x+5^{-x})^2-(5^x-5^{-x})^2(5^x-5^x)^2 & 1\\(6^x+6^{-x})^2-(6^x-6^{-x})^2(6^x-6^{-x})^2 & 1\\(7^x+7^{-x})-(7^x-7^{-x})^2(7^x-7^x)^2 & 1\end{vmatrix}$$

$$A=\begin{vmatrix}4 & (5^x-5^x)^2 & 1\\4 & (6^x-6^{-x})^2 & 1\\4 & (7^x-7^{-x})^2 & 1\end{vmatrix}$$

$$A=4\begin{vmatrix}1 & (5^x-5^x)^2 & 1\\1 & (6^x-6^{-x})^2 & 1\\1 & (7^x-7^{-x})^2 & 1\end{vmatrix}$$

$$\left[\because (a+b)^2-(a-b)^2=4ab\right]$$

$$A=C_1 \text{ and } C_3 \text{ are identical }]$$

Question48

The value of determinant $\begin{vmatrix}a-b & b+c & a\\b-a & c+a & b\\c-a & a+b & c\end{vmatrix}$ **is**

KCET 2018

Options:

A. $a^3+b^3+c^3$

B. $(a+b+c)\left[2a^2-ac-ab-bc+b^2\right]$

C. $a^3+b^3+c^3-3abc$

D. $a^3+b^3+c^3+3abc$

Answer: B

Solution:

To find the value of the determinant $\Delta=\begin{vmatrix} a-b & b+c & a \\ b-a & c+a & b \\ c-a & a+b & c \end{vmatrix}$, we can proceed with column and row operations to simplify it.

First, perform the column operation $C_3 \rightarrow C_2+C_3$:

$$\Delta=\begin{vmatrix} a-b & b+c & a+b+c \\ b-a & c+a & a+b+c \\ c-a & a+b & a+b+c \end{vmatrix}$$

Factor out $(a+b+c)$, as it's common in the third column:

$$\Delta=(a+b+c)\begin{vmatrix} a-b & b+c & 1 \\ b-a & c+a & 1 \\ c-a & a+b & 1 \end{vmatrix}$$

Next, apply the row operations $R_2 \rightarrow R_2-R_1$ and $R_3 \rightarrow R_3-R_1$:

$$\Delta=(a+b+c)\begin{vmatrix} a-b & b+c & 1 \\ 2b & a-b & 0 \\ c-2a+b & a-c & 0 \end{vmatrix}$$

Since the last column after the row operations has two zeros, the determinant simplifies to:

$$\Delta=(a+b+c)(1 \times(2b(a-c)-(a-b)(c-2a+b)))$$

Now expand the expression inside the determinant:

$$=(a+b+c)\left[2ab-2bc-\left(ac-2a^2+ab-bc+2ab-b^2\right)\right]$$

Simplify the terms:

$$=(a+b+c)\left[2ab-2bc-ac+2a^2-ab+bc-2ab+b^2\right]$$

Resulting in:

$$=(a+b+c)\left[2a^2-ac-ab-bc+b^2\right]$$

Question49

If $(x_1, y_1),(x_2, y_2)$ and (x_3, y_3) are the vertices of a triangle whose are is ' k ' square units, then $\begin{vmatrix} x_1 & y_1 & 4 \\ x_2 & y_2 & 4 \\ x_3 & y_3 & 4 \end{vmatrix}^2$ is

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Options:

A. $32k^2$

B. $16k^2$

C. $64k^2$

D. $48k^2$

Answer: C

Solution:

To solve the problem, let's start by noting that the vertices of the triangle are given as (x_1, y_1), (x_2, y_2), and (x_3, y_3) with a triangle area of k square units.

The area k of the triangle can be calculated using the determinant method as follows:

$$\frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = k$$

This implies:

$$\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = 2k$$

Next, we need to evaluate the expression:

$$\begin{vmatrix} x_1 & y_1 & 4 \\ x_2 & y_2 & 4 \\ x_3 & y_3 & 4 \end{vmatrix}^2$$

This determinant can be rewritten and expanded using properties of determinants:

$$\begin{vmatrix} x_1 & y_1 & 4 \\ x_2 & y_2 & 4 \\ x_3 & y_3 & 4 \end{vmatrix} = 4 \times \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

Thus, the determinant squared is:

$$\left(4 \cdot \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}\right)^2$$

$$= 16\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}^2$$

Substituting the value of the determinant in terms of k:

$$= 16 \cdot (2k)^2 = 16 \cdot 4k^2 = 64k^2$$

Therefore, the value of the expression is $64k^2$.

Question50

Let A be a square matrix of order 3×3, then $|5A|$ is equal to

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Options:

A. $5|A|$

B. $125|\mathrm{~A}|$

C. $25|\mathrm{~A}|$

D. $15|\mathrm{~A}|$

Answer: B

Solution:

When you multiply a square matrix by a scalar, the determinant of the new matrix is the product of the scalar raised to the power of the order of the matrix and the determinant of the original matrix. For a 3×3 matrix A, the formula is:

$|5A| = 5^3|A| = 125|A|$

Thus, the correct answer is:

Option B: $125|A|$.

Question51

$$Let\ \ \Delta = \begin{vmatrix} Ax & x^2 & 1 \\ By & y^2 & 1 \\ Cz & z^2 & 1 \end{vmatrix} \text{ and } \Delta_1 = \begin{vmatrix} A & B & C \\ x & y & z \\ zy & zx & xy \end{vmatrix}, \text{ then } \begin{vmatrix} Ax & By & Cy \\ x^2 & y^2 & z^2 \\ 1 & 1 & 1 \end{vmatrix}$$

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Options:

A. $\Delta_1 = 2\Delta$

B. $\Delta_1 = -\Delta$

C. $\Delta_1 = \Delta$

D. $\Delta_1 \neq \Delta$

Answer: C

Solution:

$$We\ have,\ \Delta=\begin{vmatrix}Ax & x^2 & 1\\ By & y^2 & 1\\ Cz & z^2 & 1\end{vmatrix} and$$

$$\Delta_1=\begin{vmatrix}A & B & C\\ x & y & z\\ zy & zx & xy\end{vmatrix}$$

$$Now,\quad \Delta_1=\begin{vmatrix}A & B & C\\ x & y & z\\ zy & zx & xy\end{vmatrix}$$

On applying
$C_1 \to xC_1, C_2 \to yC_2, C_3 \to zC_3$, we get

$$=\frac{1}{xyz}\begin{vmatrix}Ax & By & Cz\\ x^2 & y^2 & z^2\\ xyz & xyz & xyz\end{vmatrix}$$

Taking common xyz from R_3

$$=\frac{xyz}{xyz}\begin{vmatrix}Ax & By & Cz\\ x^2 & y^2 & z^2\\ 1 & 1 & 1\end{vmatrix}$$

$$=\begin{vmatrix}Ax & By & Cz\\ x^2 & y^2 & z^2\\ 1 & 1 & 1\end{vmatrix}$$

$$=\begin{vmatrix}Ax & x^2 & 1\\ By & y^2 & 1\\ Cz & z^2 & 1\end{vmatrix} [\because |A'|=|A|]$$

$$=\Delta$$

Question52

If $\begin{vmatrix}3 & x\\ x & 1\end{vmatrix}=\begin{vmatrix}3 & 2\\ 4 & 1\end{vmatrix}$, then x is equal to

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Options:

A. 4

B. 8

C. 2

D. $\pm 2\sqrt{2}$

Answer: D

Solution:

We have,

$$\begin{vmatrix} 3 & x \\ x & 1 \end{vmatrix} = \begin{vmatrix} 3 & 2 \\ 4 & 1 \end{vmatrix}$$

$\Rightarrow 3 - x^2 = 3 - 8$

$\Rightarrow x^2 = 8$

$\Rightarrow x = \pm 2\sqrt{2}$

Question53

If $2\begin{vmatrix} 1 & 3 \\ 0 & x \end{vmatrix} + \begin{vmatrix} y & 0 \\ 1 & 2 \end{vmatrix} = \begin{vmatrix} 5 & 6 \\ 1 & 8 \end{vmatrix}$, then the value of x and y are

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Options:

A. $x = 3, y = 3$

B. $x = -3, y = 3$

C. $x = 3, y = -3$

D. $x = -3, y = -3$

Answer: A

Solution:

We have,

$$2\begin{bmatrix} 1 & 3 \\ 0 & x \end{bmatrix} + \begin{bmatrix} y & 0 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 6 \\ 1 & 8 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2+y & 6 \\ 1 & 2x+2 \end{bmatrix} = \begin{bmatrix} 5 & 6 \\ 1 & 8 \end{bmatrix}$$

$\therefore \quad 2 + y = 5$ and $2x + 2 = 8$

$\Rightarrow \quad y = 3$ and $x = 3$

$\therefore \quad x = 3, y = 3$

Question54

If A is a square matrix of order 3×3, then $|KA|$ is equal to

KCET 2017

Options:

A. $K^2|A|$

B. $K|A|$

C. $3K|A|$

D. $K^3|A|$

Answer: D

Solution:

We know that,

$|KA| = K^n|A|$, where $n \times n$ is order of the matrix.

$\therefore |KA| = K^3|A|$ $\quad$ [$\because$ order of A is 3×3]

Question55

If $A = \frac{1}{\pi}\begin{vmatrix} \sin^{-1}(\pi x) & \tan^{-1}\left(\frac{x}{\pi}\right) \\ \sin^{-1}\left(\frac{x}{\pi}\right) & \cot^{-1}(\pi x) \end{vmatrix}$

$B = \begin{vmatrix} -\cos^{-1}(\pi x) & \tan^{-1}\left(\frac{x}{\pi}\right) \\ \sin^{-1}\left(\frac{x}{\pi}\right) & -\tan^{-1}(\pi x) \end{vmatrix}$,

then $A - B$ **is :**

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Options:

A. 0

B. $\frac{1}{2}$ I

C. /

D. 2!

Answer: C

Solution:

Hint

$A=\frac{1}{\pi}\left[\sin^{-1}(\pi x)\cot^{-1}(\pi x)-\tan^{-1}\left(\frac{x}{\pi}\right)\sin^{-1}\left(\frac{x}{\pi}\right)\right]$
$B=\cos^{-1}(\pi x)\tan^{-1}(\pi x)-\sin^{-1}\left(\frac{x}{\pi}\right)\tan^{-1}\left(\frac{x}{\pi}\right)$

Question56

If a matrix A is both symmetric and skew symmetric, then

KCET 2017

Options:

A. A is diagonal matrix

B. A is a zero matrix

C. A is scalar matrix

D. A is square matiox

Answer: B

Solution:

If a matrix A is both symmetric and skew-symmetric, let's analyze the implications:

A symmetric matrix satisfies the condition $A'=A$.

A skew-symmetric matrix satisfies the condition $A'=-A$.

For a matrix to be both symmetric and skew-symmetric, it must fulfill both conditions simultaneously:

$A'=A \quad \text{and} \quad A'=-A$

Comparing these, we find:

$A=-A$

This equation holds true only when every element of matrix A is zero, making A a zero matrix. Hence, if a matrix is both symmetric and skew-symmetric, it must be a zero matrix.
